6.6.7

(a) The design matrix is given by

$$X = \begin{bmatrix} 1 & 1^2 \\ 2 & 2^2 \\ 3 & 3^2 \\ 4 & 4^2 \\ 5 & 5^2 \end{bmatrix}.$$

The observation vector is given by

$$\mathbf{y} = \begin{bmatrix} 1.8 \\ 2.7 \\ 3.4 \\ 3.8 \\ 3.9 \end{bmatrix}.$$

The unknown parameter vector is

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}.$$

(b) Solving the normal equations $X^T X \hat{\boldsymbol{\beta}} = X^T \mathbf{y}$ gives the least squares solution $\hat{\boldsymbol{\beta}} = (1.76, -0.20)$.

6.7.13

Let $(\mathbf{u}, \mathbf{v}) = \mathbf{u} \cdot \mathbf{v}$. We check linearity:

$$\begin{aligned} \langle A(c\mathbf{u} + d\mathbf{v}, Aw\rangle &= (cA\mathbf{u} + dA\mathbf{v}, Aw) \\ &= c(A\mathbf{u}, Aw) + d(A\mathbf{v}, Aw) \\ &= c\langle u, w\rangle + d\langle v, w\rangle \end{aligned}$$

Symmetry:

$$\langle \mathbf{u}, \mathbf{v}\rangle = (A\mathbf{u}, A\mathbf{v}) = (A\mathbf{v}, A\mathbf{u}) = \langle \mathbf{v}, \mathbf{u}\rangle$$

Positive definiteness: $\langle \mathbf{u}, \mathbf{u}\rangle = (A\mathbf{u}, A\mathbf{u}) \geq 0$, with equality exactly when $A\mathbf{u} = 0$. Since A is invertible this is equivalent to $\mathbf{u} = 0$.

6.8.4

The two methods are the same, since in the second case the error will be exactly twice the error in the first case, and minimizing twice a quantity is the same as minimizing the quantity itself.